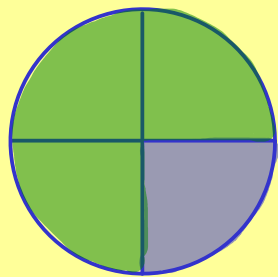




The Bright Side of Mathematics

Start Learning Numbers - Part 9

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$$



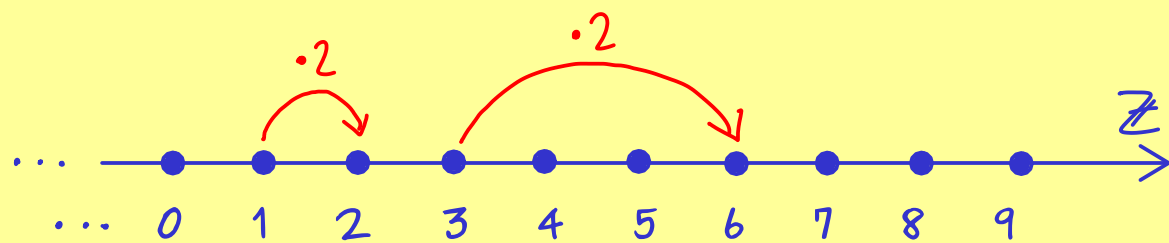
ratio: 3:1 or 3:4 or 1:4

$$\text{fraction: } \frac{3}{4} + \frac{1}{4} = 1$$

solve $4 \cdot x = 1$? \rightsquigarrow We need inverses with respect to \cdot !
Works the same as $(\mathbb{N}_0, +) \rightsquigarrow (\mathbb{Z}, +)$

For $(c, d), (a, b) \in \mathbb{Z} \times \mathbb{Z} \setminus \{0\}$ define:

$$(a, b) \sim (c, d) \quad \text{by} \quad a \cdot d = c \cdot b$$



$$(6, 3) \sim (2, 1) \quad \text{"} \frac{6}{3} = \frac{2}{1} \text{"}$$

$$\mathbb{Q} := (\mathbb{Z} \times \mathbb{Z} \setminus \{0\}) / \sim = \left\{ [(a, b)]_{\sim} \mid (a, b) \in \mathbb{Z} \times \mathbb{Z} \setminus \{0\} \right\} \quad \text{rational numbers}$$

Examples: $[(4, 2)]_{\sim} = [(6, 3)]_{\sim} = [(2, 1)]_{\sim} =: 2_{\mathbb{Q}}$

$$[(-9, -3)]_{\sim} = [(9, 3)]_{\sim} = [(3, 1)]_{\sim} =: 3_{\mathbb{Q}}, \quad [(0, 8)]_{\sim} = [(0, 1)]_{\sim} =: 0_{\mathbb{Q}}$$

$$[(-9, 3)]_{\sim} = [(-3, 1)]_{\sim} =: (-3)_{\mathbb{Q}}$$

We get all integers back!

$$[(2, 8)]_{\sim} = [(1, 4)]_{\sim} =: \left(\frac{1}{4}\right)_{\mathbb{Q}} \rightsquigarrow \text{fractions}$$

Definition: $[(a, b)]_{\sim} =: \frac{a}{b} \quad \left(\frac{2}{8} = \frac{1}{4} \right)$