The Bright Side of Mathematics

Start Learning Numbers – Part 8

$$\mathbb{Z} = \{ \ldots, (-2)_{\mathbb{Z}}, (-1)_{\mathbb{Z}}, 0_{\mathbb{Z}}, 1_{\mathbb{Z}}, 2_{\mathbb{Z}}, \ldots \}$$

$2_{\mathbb{Z}} = [(6,4)]_{\sim}$ think of "6 − 4"

think of "$(a-b)\cdot(c-d) = (ac+bd) - (ad+bc)$"

$$[(a,b)]_{\sim} \cdot [(c,d)]_{\sim} := [(a\cdot c + b\cdot d,\ a\cdot d + b\cdot c)]_{\sim}$$

The multiplication is well-defined.

map $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$

Properties of $\mathbb{Z}$ together with $\cdot$:

(a) associative

(b) commutative

(c) $1_{\mathbb{Z}} \cdot m = m$ ($1_{\mathbb{Z}}$ is neutral element)

(d) distributive

Examples:

(a) $4_{\mathbb{Z}} \cdot 2_{\mathbb{Z}} = [(4,0)]_{\sim} \cdot [(2,0)]_{\sim} = [(4\cdot 2 + 0\cdot 0,\ 4\cdot 0 + 0\cdot 2)]_{\sim} = 8_{\mathbb{Z}}$

(b) $(-4)_{\mathbb{Z}} \cdot (-2)_{\mathbb{Z}} = [(0,4)]_{\sim} \cdot [(0,2)]_{\sim} = [(0\cdot 0 + 4\cdot 2,\ 0\cdot 2 + 4\cdot 0)]_{\sim} = 8_{\mathbb{Z}}$