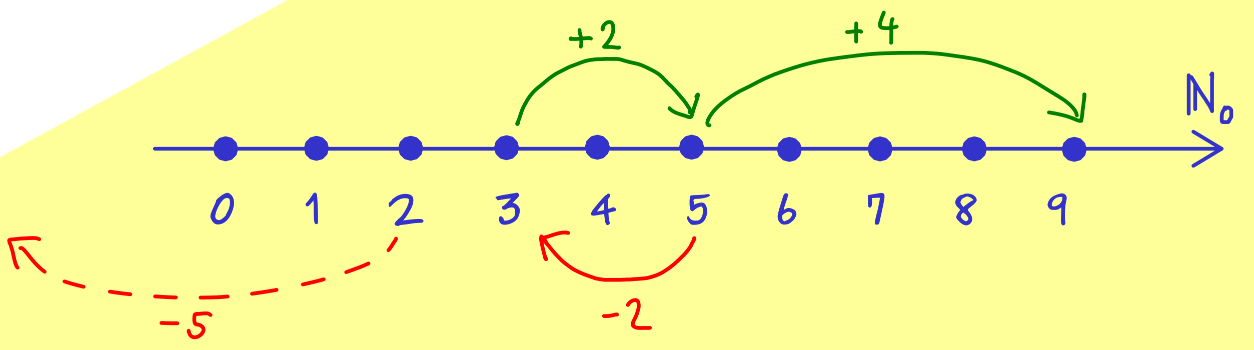
The Bright Side of Mathematics

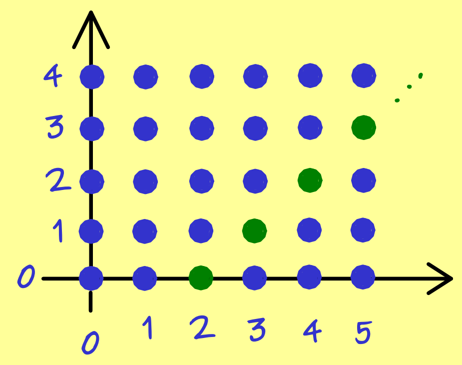
Start Learning Numbers – Part 6



$5 - 2 = 3$ $\qquad$ $2 - 5 = ?$

Idea: Look at pairs $(9,5)$, $(5,3)$, $(3,5)$

$\mathbb{N}_0 \times \mathbb{N}_0 =: \mathbb{N}_0^2$



$\vdots$
$(5,3)$
$(4,2)$
$(3,1)$
$(2,0)$
should be the "same"

$(5,3)$ stands for "$5-3$"

$(4,2)$ stands for "$4-2$"

$5-3 = 4-2$ ← not okay

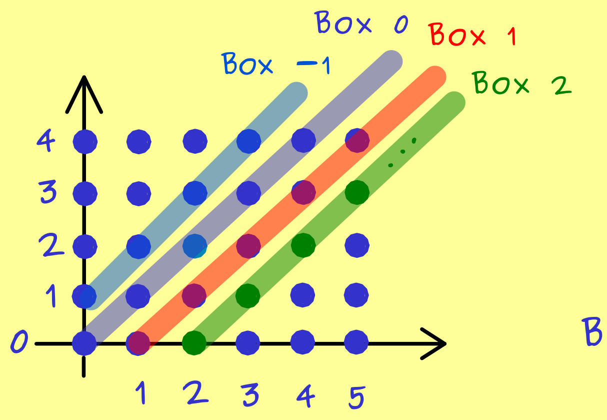
$5+2 = 4+3$ ← totally okay

$(5,3) \sim (4,2)$ (equivalent)

Equivalence relation: We write $(a,b) \sim (x,y)$ if:

$$a + y = x + b$$

Properties:
(1) $(a,b) \sim (a,b)$ (reflexive)
(2) If $(a,b) \sim (x,y)$, then $(x,y) \sim (a,b)$. (symmetric)
(3) If $(a,b) \sim (x,y)$ and $(x,y) \sim (c,d)$, then $(a,b) \sim (c,d)$. (transitive)



Property of $\mathbb{N}_0$ (cancellation):
If $m + n = \tilde{m} + n$, then $m = \tilde{m}$.

$$\text{Box } 0 = [(2,2)]_\sim := \{(x,y) \in \mathbb{N}_0^2 \mid (x,y) \sim (2,2)\}$$

is called the equivalence class of $(2,2)$.

Box 0 $= [(0,0)]_\sim = [(2,2)]_\sim$

Box 1 $= [(1,0)]_\sim = [(9,8)]_\sim$

Box 2 $= [(2,0)]_\sim$

$\vdots$

Box −1 $= [(0,1)]_\sim = [(8,9)]_\sim$

Box −2 $= [(0,2)]_\sim$

$\vdots$

$\mathbb{Z} :=$ set of all boxes (equivalence classes)