



# The Bright Side of Mathematics

## Start Learning Numbers - Part 4

Natural numbers:  $\mathbb{N}_0 = \{0, 1, 2, 3, 4, \dots\}$

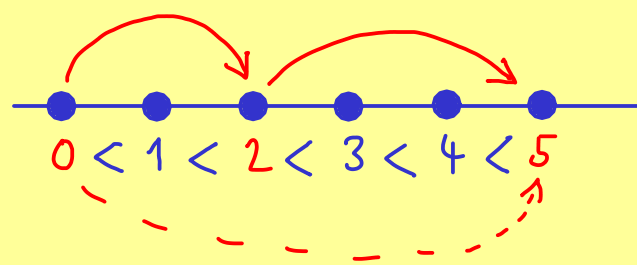
Addition  $+$  is a map  $\mathbb{N}_0 \times \mathbb{N}_0 \rightarrow \mathbb{N}_0$  with:

- $m + 0 = m$  (neutral element)
- $(k+m) + n = k + (m+n)$  (associative law)
- $m+n = n+m$  (commutative law)

### Ordering:

We write  $n \leq m$  if:

$$\exists k \in \mathbb{N}_0 : m = n + k$$



And we write  $n < m$  if:  $n \leq m \wedge n \neq m$

### Properties:

(1)  $n \leq n$  (reflexive)

(2) If  $n \leq m \wedge m \leq n$ , then  $n = m$  (antisymmetric)

(3) If  $n \leq l \wedge l \leq m$ , then  $n \leq m$  (transitive)

Proof: Assume  $n \leq l$  and  $l \leq m$  are true. So:

$$\exists k_1 \in \mathbb{N}_0 : l = n + k_1 \quad \text{and} \quad \exists k_2 \in \mathbb{N}_0 : m = l + k_2 \quad \text{are true.}$$

$$\text{Therefore: } m = l + k_2 = (n + k_1) + k_2$$

$$= n + \underbrace{(k_1 + k_2)}_{=: k \in \mathbb{N}_0} = n + k$$

Therefore:  $\exists k \in \mathbb{N}_0 : m = n + k$  is true, so  $n \leq m$  is true.