The Bright Side of Mathematics

Start Learning Numbers – Part 2

Natural numbers: $\mathbb{N}_0 = \{0, 1, 2, 3, 4, \dots\}$

Properties of $\mathbb{N}_0$:

(1) $0 \in \mathbb{N}_0$

(2) There is a map $s: \mathbb{N}_0 \to \mathbb{N}_0$ that satisfies:

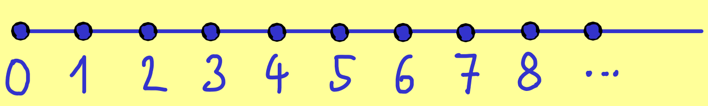
(2a) s is injective

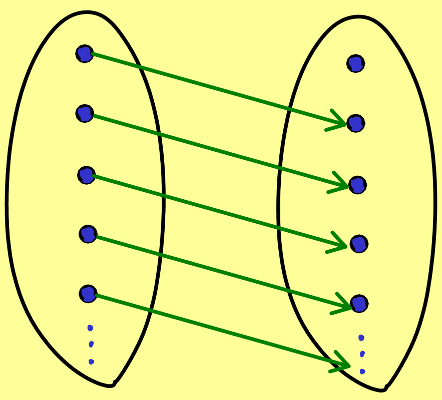
(2b) $0 \notin \operatorname{Ran}(s) = s[\mathbb{N}_0]$

(2c) If $M \subseteq \mathbb{N}_0$ with $0 \in M$ and $s[M] \subseteq M$, then $M = \mathbb{N}_0$.

(mathematical induction)

$\mathbb{N}_0$





Addition in $\mathbb{N}_0$: map $\mathbb{N}_0 \times \mathbb{N}_0 \longrightarrow \mathbb{N}_0$

$(m, n) \longmapsto m + n$

How is it defined? $2 + 4 := 6$

$\boxed{m + 0 := m}$, $m + 1 := s(m)$, $m + 2 := s(m+1)$

Recursive definition: $\boxed{m + s(n) := s(m+n)}$

$$2 + 5 = 2 + s(4) = s(2+4) = s(6) = 7$$

Dedekind's principle of recursive definition:

For a set A, $a \in A$ and $h: A \to A$, then there exists a unique map $f: \mathbb{N}_0 \to A$ with $f(0) = a$ and $f(s(n)) = h(f(n))$.

(" $a, h(a), h(h(a)), h(h(h(a))), \dots$ ")