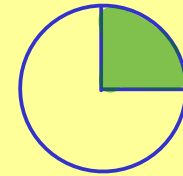




## The Bright Side of Mathematics

### Start Learning Numbers - Part 11

$$\mathbb{Q} = \left\{ \frac{a}{b} \mid a \in \mathbb{Z}, b \in \mathbb{Z} \setminus \{0\} \right\}$$


 $\leq$ 


We need to define it

Definition of  $\leq$  for  $\mathbb{Z}$ : For  $a, b \in \mathbb{Z}$ , we write  $a \leq b$  if  $\exists k \in \mathbb{N}_0 : a + k = b$

Now:  $\frac{1}{4} \leq \frac{1}{3}$  because  $3 \leq 4$

Definition of  $\leq$  for  $\mathbb{Q}$ : For  $b > 0$  and  $d > 0$

$$\frac{a}{b} \leq \frac{c}{d} \quad \text{defined by} \quad a \cdot d \leq c \cdot b$$

Properties of  $\leq$  for  $\mathbb{Q}$ : (1) Ordering: reflexive, antisymmetric and transitive.

(2) For all  $x, y, z \in \mathbb{Q}$ : If  $x \leq y$ , then  $x + z \leq y + z$

(3) For all  $x, y, z \in \mathbb{Q}$ : If  $z \geq 0$  and  $x \leq y$ , then  $x \cdot z \leq y \cdot z$

(4) Total order: For all  $x, y \in \mathbb{Q}$ , we have  $x \leq y$  or  $y \leq x$ .

(5) Archimedean property: For all  $x, \varepsilon \in \mathbb{Q}$  with  $x > 0$  and  $\varepsilon > 0$ ,

we have:  $n \in \mathbb{N}_0 : n \cdot \varepsilon = \varepsilon + \varepsilon + \varepsilon + \dots + \varepsilon > x$

