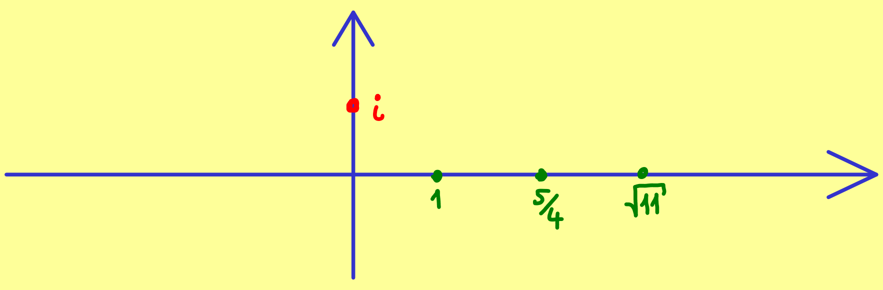
The Bright Side of Mathematics

Start Learning Complex Numbers – Part 2

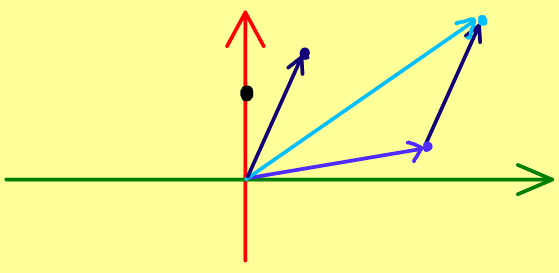


$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

$+$ addition

$\cdot$ multiplication

Addition: For $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R} \times \mathbb{R}$, we set: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} := \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix}$



Multiplication: For $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R} \times \mathbb{R}$, we set: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} := \begin{pmatrix} x_1 \cdot y_1 - x_2 \cdot y_2 \\ x_2 \cdot y_1 + x_1 \cdot y_2 \end{pmatrix}$

Why?

Short notation: $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} =: x_1 + i \cdot x_2$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0 + i \cdot 1 =: i$

Calculation: $(x_1 + i \cdot x_2) \cdot (y_1 + i \cdot y_2) \underset{\text{distributivity}}{\overset{\text{we want}}{=}} x_1 \cdot y_1 + i \cdot x_2 \cdot y_1 + i \cdot x_1 \cdot y_2 + \underset{\text{we want}\ -1}{i^2} \cdot x_2 \cdot y_2$

$= (x_1 \cdot y_1 - x_2 \cdot y_2) + i \cdot (x_2 \cdot y_1 + x_1 \cdot y_2)$

Check: $i^2 = i \cdot i = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 - 1 \\ 0 + 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} = -1 + i \cdot 0 = -1$

Properties:
- We write $\mathbb{C} := \mathbb{R}^2$ when we have $+$ and $\cdot$ from above.

field:
- $(\mathbb{C}, +, \underset{= 0 + i \cdot 0}{0})$ is an abelian group (commutative, associative, neutral element, inverses)
- $(\mathbb{C} \setminus \{0\}, \cdot, \underset{= 1 + i \cdot 0}{1})$ is an abelian group (commutative, associative, neutral element, inverses)
- distributive law
- no nice ordering $\leq$ like for $\mathbb{R}$