The Bright Side of Mathematics

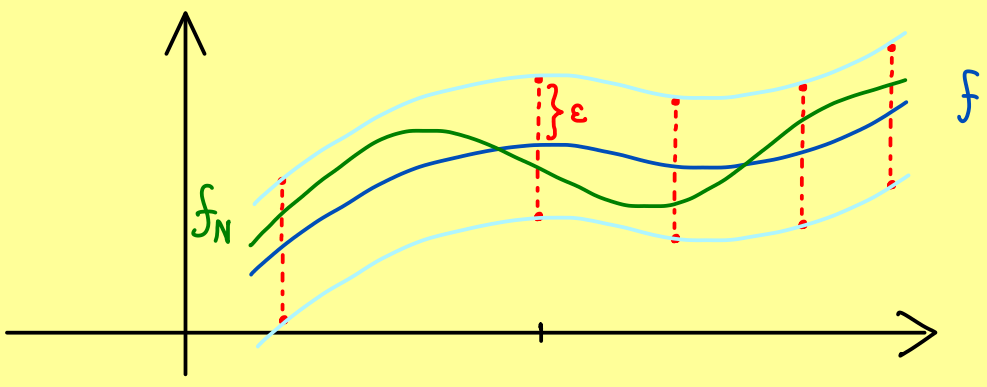
Real Analysis – Part 25

$(f_1, f_2, f_3, f_4, f_5, \dots)$ is pointwisely convergent to $f: I \longrightarrow \mathbb{R}$

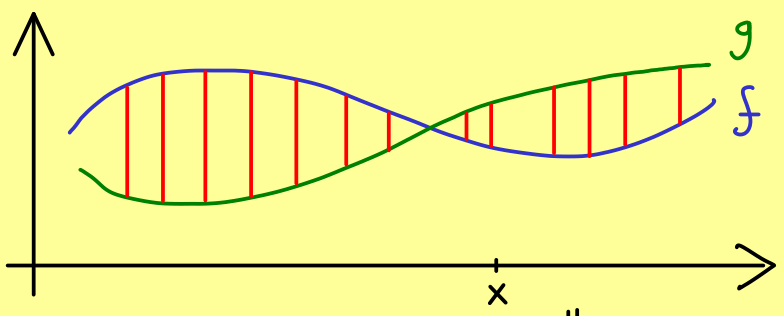
$$\forall \tilde{x} \in I \quad \forall \varepsilon > 0 \quad \exists N_{\tilde{x}} \in \mathbb{N} \quad \forall n \geq N: \quad |f_n(\tilde{x}) - f(\tilde{x})| < \varepsilon$$

Definition: $(f_1, f_2, f_3, f_4, f_5, \dots)$ is uniformly convergent to $f: I \longrightarrow \mathbb{R}$ if

$$\forall \varepsilon > 0 \quad \exists N \in \mathbb{N} \quad \forall n \geq N \quad \forall \tilde{x} \in I: \quad |f_n(\tilde{x}) - f(\tilde{x})| < \varepsilon$$



Distance for functions:



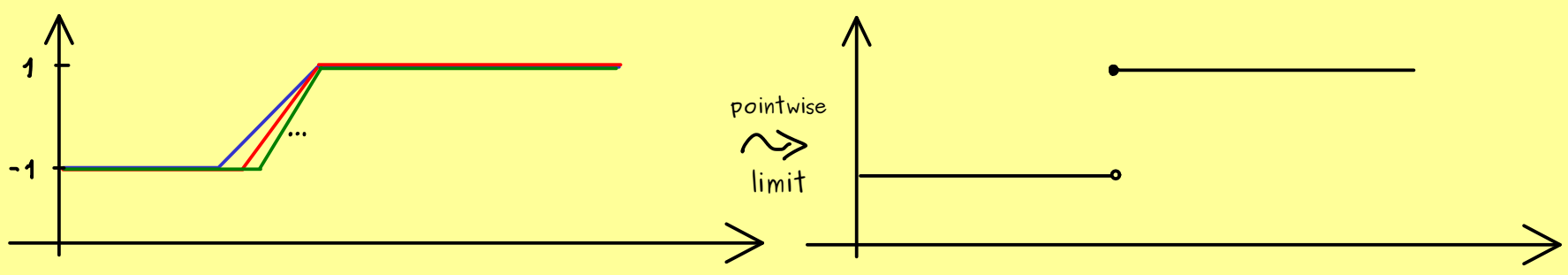
$f: I \longrightarrow \mathbb{R}$

$g: I \longrightarrow \mathbb{R}$

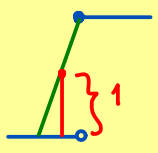
supremum norm of $f - g$ $\longrightarrow$ $\|f - g\|_\infty = \sup_{x \in I} |f(x) - g(x)|$

Uniform convergence means: $\|f_n - f\|_\infty \xrightarrow{n \to \infty} 0$

Example:



$\|f_n - f\|_\infty \geq 1$ for all n



Result pointwise convergence $\not\Rightarrow$ uniform convergence

$\Leftarrow$