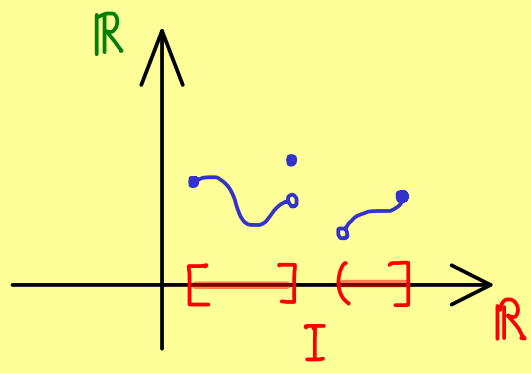
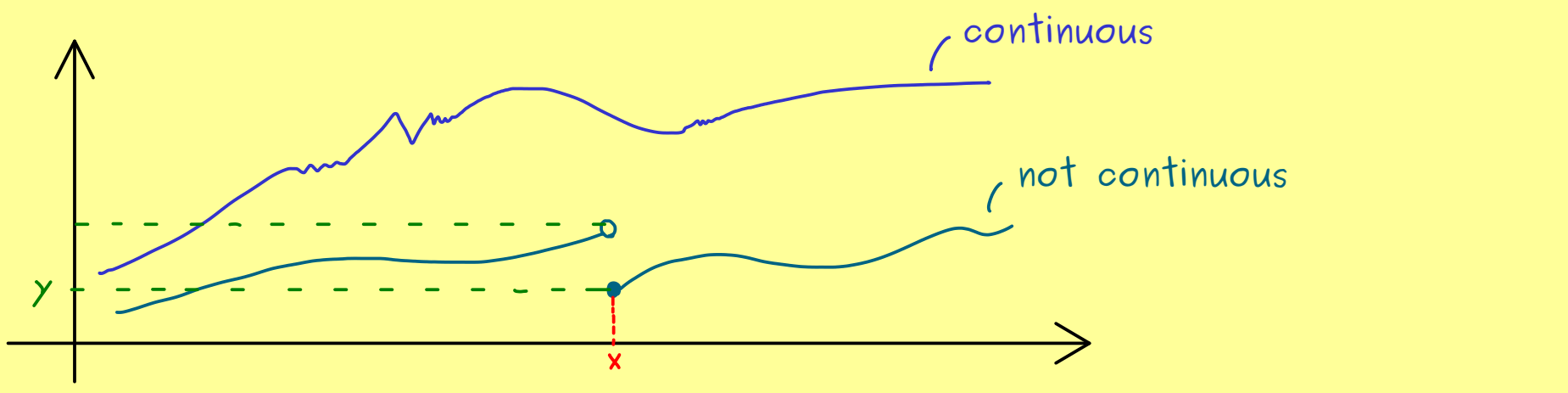
The Bright Side of Mathematics

Real Analysis – Part 23

Function: $f: I \longrightarrow \mathbb{R} \quad (I \subseteq \mathbb{R})$



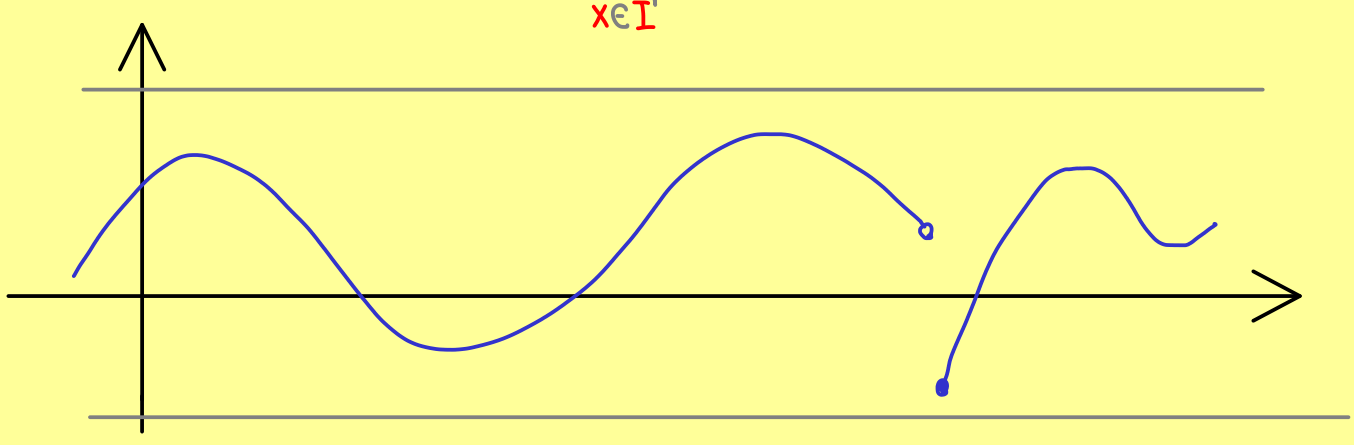
Later: continuous functions $f: \mathbb{R} \longrightarrow \mathbb{R}$



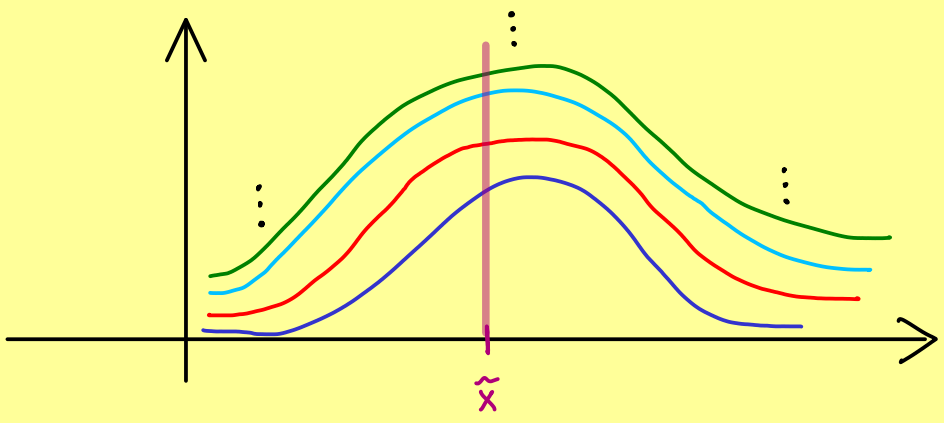
Definition: $f: I \longrightarrow \mathbb{R}$ is called a bounded function if

$$\{ f(x) \mid x \in I \} = \mathrm{Ran}(f) = f[I] \text{ is a bounded set in } \mathbb{R}$$

$$\left(\Leftrightarrow \sup_{x \in I} |f(x)| < \infty \right)$$



Sequence of functions:



For any fixed $\tilde{x} \in I$,
we get an ordinary sequence of real numbers:

$$(f_1(\tilde{x}), f_2(\tilde{x}), f_3(\tilde{x}), f_4(\tilde{x}), f_5(\tilde{x}), \dots)$$

sequence:

$$(f_1, f_2, f_3, f_4, f_5, \dots)$$

with sequence members:

$f_1: I \longrightarrow \mathbb{R}$

$f_2: I \longrightarrow \mathbb{R}$

$f_3: I \longrightarrow \mathbb{R}$

$\vdots$