A4.2

$$\frac{1}{2}(\ln(z))^2 + (\sqrt{3}-j)\ln(z) + 4 + 2\sqrt{3}j = 0$$

Erkenne: Quadratische Gleichung in $\ln(z) =: \gamma$

$$\frac{1}{2}\gamma^2 + (\sqrt{3}-j)\gamma + 4 + 2\sqrt{3}j = 0 \qquad | \cdot 2$$

$$\Leftrightarrow \quad \gamma^2 + 2(\sqrt{3}-j)\gamma = -8 - 4\sqrt{3}j$$

(Quadr. Ergänzung)

$$\Leftrightarrow \quad (\gamma + (\sqrt{3}-j))^2 = -8 - 4\sqrt{3}j + (\sqrt{3}-j)^2$$

$$\Leftrightarrow \quad (\gamma + (\sqrt{3}-j))^2 = -6(1+\sqrt{3}j)$$

$$\Leftrightarrow \quad (\gamma + (\sqrt{3}-j))^2 = 12e^{-\frac{2}{3}\pi j + 2\pi jk} \qquad k \in \mathbb{Z}$$

$$\Leftrightarrow \quad \begin{cases} \gamma_0 + \sqrt{3} - j = \sqrt{12}\, e^{-\frac{1}{3}\pi j} \\ \gamma_1 + \sqrt{3} - j = \sqrt{12}\, e^{-\frac{1}{3}\pi j + \pi j} \end{cases}$$

$$\Leftrightarrow \quad \begin{cases} \gamma_0 = -\sqrt{3} + j + \sqrt{12}\, e^{-\frac{1}{3}\pi j} \\ \gamma_1 = -\sqrt{3} + j + \sqrt{12}\, e^{2/3\pi j} \end{cases}$$

$$\Leftrightarrow \begin{cases} \gamma_0 = -\sqrt{3} + j + \overbrace{\sqrt{12}}^{2\sqrt{3}}\left(\frac{1}{2} - \frac{1}{2}\sqrt{3}j\right) \\ \gamma_1 = -\sqrt{3} + j + \sqrt{12}\left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}j\right) \end{cases}$$

$$\Leftrightarrow \begin{cases} \gamma_0 = -\sqrt{3} + j + \sqrt{3} - 3j \\ \gamma_1 = -\sqrt{3} + j - \sqrt{3} + 3j \end{cases}$$

$$\Leftrightarrow \begin{cases} \gamma_0 = -2j \\ \gamma_1 = -2\sqrt{3} + 4j \end{cases}$$