Lösung

(a) $\int x^2 \cos(x)\, dx$ mit partieller Integration

$u = x^2 \qquad u' = 2x$

$v' = \cos(x) \qquad v = \sin(x)$

$$= \left[x^2 \sin(x)\right] - \int 2x \sin(x)\, dx$$

nochmals: $u = 2x \qquad u' = 2$

$v' = \sin(x) \qquad v = -\cos(x)$

$$= \left[x^2 \sin(x)\right] + \left[2x \cos(x)\right] - \int 2 \cos(x)\, dx$$

$$= \left[x^2 \sin(x) + 2x \cos(x)\right] - 2 \sin(x) + C$$

$$= \underline{(x^2 - 2) \sin(x) + 2x \cos(x) + C} \quad , \quad C \in \mathbb{R}$$

(b) $\int e^{-x} \cos(5x)\, dx$ wieder zweimal partiell integrieren:

$$= \left[-e^{-x} \cos(5x) + 5e^{-x} \sin(5x)\right] - 25 \int e^{-x} \cos(5x)\, dx$$

gleiches Integral!

$$\Rightarrow 26 \int e^{-x} \cos(5x)\, dx = -e^{-x} \cos(5x) + 5e^{-x} \sin(5x) + C$$

$$\Rightarrow \int e^{-x} \cos(5x)\, dx = \underline{\frac{1}{26}\left(-e^{-x} \cos(5x) + 5e^{-x} \sin(5x)\right) + C}$$

$C \in \mathbb{R}$

(c) $\displaystyle\int \frac{1}{x(1+x)(1+x+x^2)}\,dx$ Partialbruchzerlegung!

<u>Ansatz:</u> $\displaystyle\frac{1}{x(1+x)(1+x+x^2)} = \frac{A}{x} + \frac{B}{1+x} + \frac{Cx+D}{1+x+x^2}$ ↙ für quadr. Term!

Multiplizieren mit Hauptnenner:

$$1 = A(x+1)(x^2+x+1) + Bx(x^2+x+1) + (Cx+D)x(x+1)$$

Setze passende x-Werte ein:

$x=0 \Rightarrow \underline{A=1}$

$x=-1 \Rightarrow \underline{B=-1}$

Oder Koeffizientenvergleich:

Koeffizient v. x^3: $0 = A+B+C \Rightarrow \underline{C=0}$

Koeffizient v. x^2: $0 = 2A+B+C+D \Rightarrow \underline{D=-1}$

Also:

$$\int f(x)\,dx = \int \left(\frac{1}{x} - \frac{1}{x+1} - \underbrace{\frac{1}{x^2+x+1}}_{\leftarrow(*)}\right)dx$$

$$= \ln|x| - \ln|x+1| - \frac{2}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right) + C,\quad C\in\mathbb{R}$$

(*)
(Hinweis: Schreibe $x^2+x+1 = (x+\frac{1}{2})^2 + \frac{3}{4} = \left[\left(\frac{2x+1}{\sqrt{3}}\right)^2+1\right]\cdot\frac{3}{4}$)

(d) $$\int_0^{\pi/2} \sin(x)\cos(x)\,dx$$ Partiell oder mit Substitution!

$$= \int_0^{\pi/2} u(x)\cdot u'(x)\,dx \qquad \text{mit} \quad u(x) = \sin(x)$$

$$= \int_{\sin(0)}^{\sin(\pi/2)} u\,du = \left[\frac{1}{2}u^2\right]_0^1 = \underline{\frac{1}{2}}$$

(e) $$\int_2^3 \frac{1}{x\sqrt{x^2-1}}\,dx$$ Substitution $u = \sqrt{x^2-1}$

$$\Rightarrow x^2 = u^2+1$$

$$u' = \frac{du}{dx} = \frac{x}{\sqrt{x^2-1}} = \frac{x}{u}$$

$$= \int_{\sqrt{4-1}}^{\sqrt{9-1}} \frac{1}{x\cdot u}\cdot\frac{u}{x}\cdot du = \int_{\sqrt{3}}^{\sqrt{8}} \frac{1}{x^2}\,du$$

$$= \int_{\sqrt{3}}^{\sqrt{8}} \frac{1}{u^2+1}\,du = \underline{\arctan(\sqrt{8}) - \arctan(\sqrt{3})}$$