

Lösung 4.3

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & -1 & 2 \end{pmatrix}, \quad p_A(\lambda) = (1-\lambda)^2(2-\lambda) \quad \begin{array}{l} \lambda_1 = 1 \\ \lambda_2 = 2 \end{array}$$

$$\underline{\lambda_2 = 2}: \quad \text{EV } \underline{u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}$$

$$\underline{\lambda_1 = 1}: \quad$$

$$\text{Kern}(A - 1 \mathbb{1}) = \text{Kern} \begin{pmatrix} 1 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \end{pmatrix} = \text{span} \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

"Ein Vektor zu wenig, also:"

$$\text{Kern}(A - 1 \mathbb{1})^2 = \text{Kern} \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \text{span} \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

↙
Hauptvektor 2. Stufe

$$\text{Setze } \underline{v_2 := \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}$$

$$\text{Dann } (A - 1 \mathbb{1}) v_2 = \underline{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} =: v_1}$$

$$T = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$v_1 \quad v_2 \quad u_1$

$$\text{Also: } J_A = \underline{T^{-1} A T = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}}$$

$$B = \begin{pmatrix} 3 & 0 & -1 & 0 \\ 2 & 3 & -2 & -1 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & -1 & 2 \end{pmatrix}$$

$$p_B(\lambda) = (3-\lambda)^2(2-\lambda)^2$$

$$\lambda_1 = 2$$

$$\lambda_2 = 3$$

$$\underline{\lambda_1 = 2}$$

$$\text{Kern}(A - 2\mathbb{1}) = \text{span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\Rightarrow \text{Eigenraum genau richtig groß. Setze } u_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, u_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\underline{\lambda_2 = 3}$$

$$\text{Kern}(A - 3\mathbb{1}) = \text{span} \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

\Rightarrow "Ein Vektor zu wenig!" Also:

$$\text{Kern}(A - 3\mathbb{1})^2 = \text{span} \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Hauptvektor 2. Stufe

$$\text{Setze } v_2 := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \text{ und } v_1 := (A - 3\mathbb{1})v_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{Setze } T := (v_1, v_2, u_1, u_2)$$

$$J_B = T^{-1} B T = \begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$