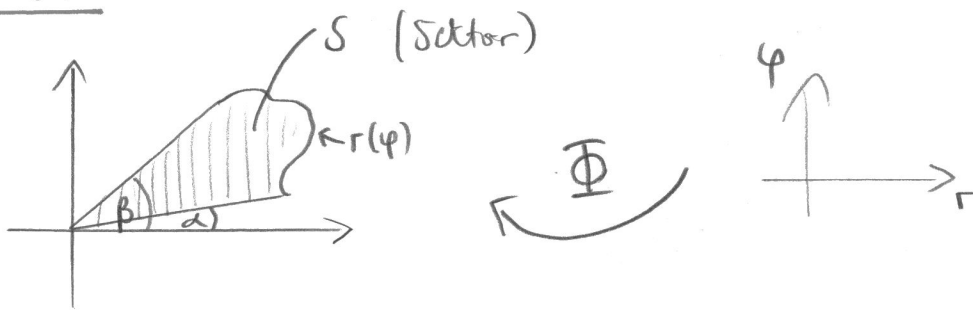


44 (b)



Polarkoordinaten: $\Phi(r, \varphi) = r \cdot \begin{pmatrix} \cos \varphi \\ \sin \varphi \end{pmatrix}$

$\Phi: U \rightarrow S$ mit $U := \{(r, \varphi) \mid \alpha < \varphi < \beta, 0 < r < r(\varphi)\}$

Flächeninhalt:

Transformationsformel

$$\begin{aligned} A(S) &= \int_{\Phi(U)} dx dy \stackrel{\downarrow}{=} \int_U |\det J_{\Phi}(r, \varphi)| dr d\varphi = \int_{\alpha}^{\beta} \int_0^{r(\varphi)} r \cdot dr d\varphi \\ &= \frac{1}{2} \int_{\alpha}^{\beta} (r(\varphi))^2 d\varphi \end{aligned}$$

Bsp: $r(\varphi) = |\sin(\frac{\pi}{2}\varphi)|$, $\beta = 2\pi$, $\alpha = 0$

$\Rightarrow A(S) = \frac{\pi}{2}$ (z.B. mit $2\sin^2(\frac{x}{2}) = 1 - \cos(x)$)