

Problem 2.4.1

We form the set $A = \left\{ x \in \mathbb{R} \mid \frac{3}{|x-9|} > \frac{2}{x+2} \right\}$ to an union of intervals:

$$(1) \quad x - 9 > 0$$

$$\Rightarrow \frac{3}{x-9} > \frac{2}{x+2} \Rightarrow 3x + 6 > 2x - 18 \Rightarrow x > -24 \Rightarrow x \in (9, \infty)$$

$$(2) \quad x - 9 < 0 \text{ and } x + 2 > 0$$

$$\Rightarrow \frac{3}{-(x-9)} > \frac{2}{x+2} \Rightarrow 3x + 6 > -2x + 18 \Rightarrow x > \frac{12}{5} \Rightarrow x \in \left(\frac{12}{5}, 9\right)$$

$$(3) \quad x - 9 < 0 \text{ and } x + 2 < 0$$

$$\Rightarrow \frac{3}{-(x-9)} > \frac{2}{x+2} \Rightarrow 3x + 6 < -2x + 18 \Rightarrow x < \frac{12}{5} \Rightarrow x \in (-\infty, -2)$$

Problem 2.4.3

Let K be an ordered field, $x, y \in K$ and $x < y$. Since $\frac{1}{2}(x - y) < 0$ and $\frac{1}{2}(y - x) > 0$, we can choose $z = x + \frac{1}{2}(y - x)$ and get $x < z < y$. \square